

Globethics Repository



FITCH' S PARADOX AND THE PROBLEM OF SHARED CONTENT

This page was generated automatically upon download from the Globethics Repository.
More information on Globethics see <https://www.globethics.net>. Data and content policy
of Globethics Repository see <https://repository.globethics.net/pages/policy>.

Item Type	Article
Authors	Sander, Thorsten
Publisher	Abstracta
Rights	With permission of the license/copyright holder
Download date	2026-07-13 22:03:37
Link to Item	http://hdl.handle.net/20.500.12424/172198

FITCH'S PARADOX AND THE PROBLEM OF SHARED CONTENT

Thorsten Sander

Abstract

According to the “paradox of knowability”, the moderate thesis that (necessarily) all truths are *knowable* – ‘ $\forall p (p \supset \Diamond Kp)$ ’ – implies the seemingly preposterous claim that all truths are actually *known* – ‘ $\forall p (p \supset Kp)$ ’ –, i.e. that we are omniscient. If Fitch’s argument were successful, it would amount to a knockdown rebuttal of anti-realism by reductio.

In the paper I defend the nowadays rather neglected strategy of intuitionistic revisionism. Employing only intuitionistically acceptable rules of inference, the conclusion of the argument is, firstly, not ‘ $\forall p (p \supset Kp)$ ’, but ‘ $\forall p (p \supset \neg\neg Kp)$ ’. Secondly, even if there were an intuitionistically acceptable proof of ‘ $\forall p (p \supset Kp)$ ’, i.e. an argument based on a different set of premises, the conclusion would have to be interpreted in accordance with Heyting semantics, and read in this way, the apparently preposterous conclusion would be true on conceptual grounds and acceptable even from a realist point of view.

Fitch’s argument, understood as an immanent critique of verificationism, fails because in a debate dealing with the justification of deduction there can be no interpreted formal language on which realists and anti-realists could agree. Thus, the underlying problem is that a satisfactory solution to the “problem of shared content” is not available.

I conclude with some remarks on the proposals by J. Salerno and N. Tennant to reconstruct certain arguments in the debate on anti-realism by establishing aporias.

In 1963 Frederic Fitch presented an argument that is now commonly regarded as one of the most promising arguments against anti-realism or “verificationism”. If, for sake of simplicity, we take Fitch’s highly plausible ‘theorem 1’ (1963: 138)

$$(T1) \neg\Diamond K(p \ \& \ \neg Kp)$$

for granted, one can show in a straightforward manner that the distinctive thesis of semantic anti-realism, namely the principle of knowability

$$(PK) \forall p (p \supset \Diamond Kp)$$

leads to a rather implausible result. As the principle of knowability is supposed to be universally valid, we may infer:

$$(PK^*) (p \& \neg Kp) \supset \Diamond K(p \& \neg Kp).$$

But together with (T1), (PK*) implies, by modus tollens and universal introduction, a result that might be labelled a “principle of omniscience”:

$$(PO) \forall p \neg(p \& \neg Kp)$$

The comparatively modest thesis that all truths are *knowable* thus seems to imply the preposterous result that there is no true proposition that would not be *known*.

Fitch's argument surely deserves to be called a *paradox* of knowability: it is based on some plausible principles that, by employing reasonable rules of inference, lead to an apparently implausible result, and as in the case of other paradoxes, there are altogether three ways of circumventing the problem. First, one might try to reject one of the premises the argument is based on. If one does not wish to dismiss outright the principle of knowability¹, this type of strategy leads to the common anti-realistic proposal to restrict (PK) to a class of ‘basic’ or ‘Cartesian’ propositions.²

A second possible remedy consists in using a weaker logic than classical logic. Employing only intuitionistically acceptable rules of inference, we can infer from (PO)

$$(PO_1) \neg \exists p (p \& \neg Kp)$$

as well as

$$(PO_2) \forall p (p \supset \neg \neg Kp),$$

but not

$$(PO_3) \forall p (p \supset Kp).$$

¹ See e.g. Melia 1991

² See Dummett 2001; Tennant 1997: 272-276

If, however, there is any reason to reject (PO_3) , its intuitionistic counterparts (PO_1) , (PO_2) or (PO) may not be any more plausible.

The third means of dealing with paradoxes – accepting the seemingly false conclusion, in this case (PO_3) – does not seem to be tenable in the case of Fitch's argument, as it is widely accepted that embracing (PO_3) without further qualifications would amount to a “preposterous” (Hart 1979: 156) form of verificationism or to a “radical idealism” (Melia 1991: 341). Claims like these, however, are based on the assumption that the only correct way of translating (PO_3) into ordinary language would be something like “we are omniscient” or “all truths are known”. The essential aim of the following considerations is to show that this assumption is highly questionable.

As most anti-realists tend to embrace a kind of logical revisionism³ the divergent interpretations of logical vocabulary (in this case especially implication and negation) pose a genuine problem for the advocate of Fitch's argument.⁴ By this, I do not mean the fact – already mentioned – that intuitionistic logic just permits us to deduce (PO) , (PO_1) and (PO_2) but not (PO_3) . Here it is crucial to remember that an intuitionistic way of thinking is not only characterized by dropping the rule of double negation ‘ $\neg\neg p \Rightarrow p$ ’ in systems of natural deduction, but furthermore by the idea that (thereby) the meanings of logical connectives are substantially changed, and if it is doubtful whether (PK) actually commits anti-realists to an absurd claim this change of meaning naturally has to be taken into account.

A short comparison may help to clarify the point. If the controversial principle *tertium non datur* ‘ $p \vee \neg p$ ’ is interpreted according to the semantics of classical logic it is virtually impossible to understand why intuitionists reject that (classical) theorem. But on the basis of the intuitionistic Brouwer-Heyting-Kolmogorov (BHK) interpretation of logical connectives, ‘ $p \vee \neg p$ ’ does not mean ‘ p is the case, or not- p is the case’, but rather ‘either we can prove that p is the case, or we can prove that p implies an absurdity’. Translating ‘ $p \vee \neg p$ ’ with BHK yields a substantial philosophical thesis S. Shapiro (1993: 283) labels (Gödelian) ‘Optimism’ and which, following N.

³ It is matter of dispute whether a non-revisionary anti-realism would be a coherent position (see Wright 1987: 317-341; Rasmussen & Ravnkilde 1982). In any case, Fitch's paradox should give anti-realists a good reason not to embrace classical logic. This point is also stressed by Williamson 1992: 63.

⁴ For the relation between intuitionism and the paradox of knowability in general see Hart 1979: 164-165; Williamson 1982; Williamson 1988; Williamson 1992; Wright 1987: 309-316.

Tennant's (2000: 831) proposal, may be formalised in the following way: ' $\forall p (\Diamond Kp \vee \Diamond K\neg p)$ '.

The BHK interpretation is, admittedly, informal and quite elusive⁵, but undoubtedly it is *the intended interpretation* of intuitionistic calculi⁶; any proposal for a formal semantics of intuitionistic logic is expected to conform to that intuitive interpretation. So let us see what the seemingly preposterous claim (PO₃) means against that background and how anti-realists who accept intuitionistic logic – most anti-realists, that is – should accordingly deal with Fitch's argument. Intuitionistic implications ' $p \supset q$ ' are, of course, not to be rendered as 'if p is the case, then q is the case', but roughly as 'if there is a proof of p it can be transformed into a proof of q'.⁷ Applied to (PO₃) we then get the statement 'if there is a proof of p it is known (at some point of time) that there is a proof of p', and this is arguably, *even for realists*, not a preposterous thesis at all, but rather an analytic statement or a conceptual truth: if somebody has in fact managed to show that p is true, he necessarily knows that p (as a proof is a way of coming to know that something is the case) and, according to the usual definition of the operator 'K', p would then also be known.

From a strictly intuitionistic point of view (PO₃), therefore, does not embody the audacious claim that we are omniscient; it simply states that proven truths are proven knowledge. Thus, even if (PK) in intuitionistic logic actually implied not only (PO₁), (PO₂) or (PO), but also (PO₃), this would not amount to a refutation of verificationism, as the conclusion would then be equivalent to a thesis on which realists and anti-realists could agree.⁸

The claim that intuitionists are *obliged* to accept (PO₃), just because of the meaning they give to implications, is not at all new, but very often authors who realise this semantic obligation fail to draw the right consequences with respect to Fitch's argument. In his *Elements of Intuitionism* as well as in his paper "The Philosophical Basis of Intuitionistic Logic", M. Dummett vindicates a claim roughly equivalent to

⁵ See van Dalen 1986: 243.

⁶ See e.g. Artemov 2001: 1.

⁷ See e.g. Heyting 1971: 102-103

⁸ The claim that (PO₃) is a rather trivial observation is embraced by Rasmussen & Ravnkilde (1982: 436-437) and by Martino & Usberti (1994: 90).

(PO₃)⁹, but in a more recent paper (2001) thinks it nevertheless necessary to restrict (PK) to a class of 'basic statements'. W. D. Hart, on the other hand, devises an argument quite similar to the one presented above, but considers it to be a refutation of intuitionism:

For being in an intuitionist position to assert that $(\forall x) (Fx \rightarrow Gx)$ requires a method which, given an object and a proof that it is F, yields a proof that it is G. In the present instance this means: suppose we are given a sentence [...] and a proof that it is true. Read the proof; thereby you come to know that the sentence is true. Reflecting on your recent learning, you recognize that the sentence is now known by you; this shows that the truth is known. If this argument is intuitionistically acceptable [...], then I think that fact reflects poorly on intuitionism; surely we have good inductive grounds for believing that there are truths as yet unknown. (Hart 1979: 165)

There is, however, a serious flaw in this argument. Hart starts by taking the intuitionistic interpretation of logical vocabulary for granted and correctly concludes that constructivists have to accept (PO₃). But then he goes on to *re-interpret* (PO₃) in a classical manner. Interpreted classically, the principle indeed means that all (realistically conceived) truths are known, but intuitionistically it only says that all proven truths are provably known. Thus Hart simply confuses two distinct meanings of the horseshoe; the allegedly uncontroversial assertion 'that there are truths as yet unknown' is compatible with a constructivist reading of (PO₃).

But is it really true that the BHK interpretation of the horseshoe *forces* the intuitionist to accept (PO₃) as a conceptual truth? In two of his papers, T. Williamson (1982; 1988) tries to show that (PO₃) is not 'disastrously provable for intuitionists' (1982: 207) by distinguishing between proof *types* and proof *tokens*. Williamson concedes that 'a proof token of P (in a mind, on paper) can be turned into a proof token that P is known', but denies that 'every proof type (as the permanent possibility of a token) can be turned into a proof type that P is known' (1982: 206-207). It has already been pointed out that if (PO₃) is semantically valid according to the BHK interpretation, then this should not lead to a "disastrous" result; on the basis of Williamson's

⁹ The seemingly absurd thesis Dummett tries to defend is ' $A \rightarrow \exists n (\vdash_n A)$ ' (interpreted classically: 'A true proposition is proved at a certain point of time'). See Dummett 1977: 348-9; Dummett 1973: 233-237.

distinction one may, however, raise the doubt whether (PO_3) is in fact provable for intuitionists.

The point at issue is, of course, how the locution 'there is a proof of p ' is to be understood. It is quite clear that (PO_3) would *not* be valid if proofs had to be regarded as denizens of a Fregean third realm: if a proof of, say, Goldbach's conjecture existed in such a realm, there would be no guarantee that the proof will ever be discovered by us, and in that sense, the existence of a proof of p would not imply the existence of a proof that p is known. But it is equally clear that the intuitionist should not be ready to accept the claim that proofs have a kind of existence apart from our (possible) cognitions of them. Dummett rightly points out that such a realist notion of proof is not only much less plausible than a realist view of mathematical entities, for it seems to belong to the very notion of proof that a proof is the result of our cognitive endeavours. Furthermore, such a notion seems to make it virtually impossible to criticize mathematical platonism:

But if we admit such a conception of proofs, we can have no objection to a parallel conception of mathematical objects such as natural numbers, real numbers, metric spaces, etc.; and then we shall have no motivation for abandoning a realistic, that is, platonist, interpretation of mathematical statements in the first place. (Dummett 1982: 258-259)¹⁰

According to Williamson, however, we do not have to appeal to a 'suspiciously Platonist' (1988: 430) notion of proofs or proof types in order to show that (PO_3) does not hold intuitionistically; all we need is the parsimonious and ontologically neutral concept of 'sameness in type': to speak of 'proof-types is then just to speak of proof-tokens in ways not sensitive to differences between proof-tokens of the same type; there need be no suggestion that the existence of a type is anything over and above the existence of tokens of that type' (Williamson 1988: 430). Williamson concedes that the distinction between proof types and proof tokens does not make any real difference with respect to (PO_3) if a proof token for a proposition p is actually known; if, however, p has not been proved (as in the case of Goldbach's conjecture) he assumes the situation to be different:

¹⁰ D. Prawitz, whose conception of provability is criticized in the quoted passage, actually advocates a rather platonist or "possibilist" reading of 'there is a proof'. For the controversy between Prawitz and Dummett concerning this matter see also Raatikainen (2004: 135-141) and Cozzo (1994: 74-77).

If P has not yet been decided, however, the best we can do is to consider the function f itself, for the hypothetical proof-token of P is the only one we have to play with in attempting to construct a proof-token of KP . But f is not a unitype-function: for there can be proof-tokens p and q of P of the same type but carried out at different times, in which case $f(p)$ and $f(q)$ are proof-tokens of distinct types, for their conclusions differ: $f(p)$ is a proof that P is proved at $t(p)$, $f(q)$ is a proof that P is proved at $t(q)$, and $t(p) \neq t(q)$. Thus if we have no decision procedure for P , we have no way of constructing a proof of $P \rightarrow KP$ [...].¹¹

Martino and Usberti (1994) have already shown that this argument of Williamson is problematic in two respects. So it should be sufficient to briefly restate their objections. Firstly, Williamson's approach just does not seem faithful to the spirit of intuitionistic logic.¹² From the viewpoint of orthodox intuitionism, a proof of an implication consist in a intuitive knowledge of how to transform a proof of the antecedent into a proof of the consequent, and such a knowledge *does not have to* involve any function from proofs to proofs (Martino & Usberti 1994: 91). Secondly, even if one grants that an intuitionistic proof of an implication can be regarded as a function, one cannot expect such a function f to operate on *hypothetical* proof tokens:

Its arguments cannot be but *given* proof-tokens; as long as no proof of A is known, f has nothing to map. So we can still define f as the constant function which, once a proof p of A is known, maps every proof q of A into the proof that K_0A is known at time $t(p)$.¹³

Additionally, one should stress that Williamson's proposal can be attractive only to intuitionists who are anxious to reject (PO_3) . But why *should* an intuitionist wish to do so, especially if dismissing (PO_3) would require us to interpret intuitionistic implications in a fairly unorthodox way? Thus, it should be safe to conclude that on any reasonable intuitionistic interpretation of the horseshoe (PO_3) should in fact be a conceptual truth.

¹¹ Williamson 1988: 430-431. In his *Knowledge and its Limits* Williamson offers a different argument for the thesis that the existence of a proof of an assertion 'does not imply that one is in a position to know that one has a proof of it' (2000: 110-113). As this claim is based on his complex overall argument against "luminosity" I cannot discuss it here.

¹² Williamson (1988: 429) explicitly concedes that is 'very hard to avoid' (PO_3) if a semantic account of the conditional is given in the common intuitionistic manner.

¹³ Martino & Usberti 1994: 91. ' K_0 ' means here the same as the usual operator ' K ' ("it is known or will be known").

The preceding remarks concerned only the question of how to translate (PO₃) or (PK) into ordinary English if we accept the background of intuitionistic logic. But what about the opposite task? Is there any way of formalizing the *ordinary English sentence*

(PK_e) All truths are knowable

available to the intuitionist? (PK_e), after all, seems to be a *substantial* philosophical thesis, whereas an intuitionistic reading of (PK) yields a conceptual truth, for even the stronger claim (PO₃) is semantically valid according to the BHK interpretation. Is the intuitionist obliged to declare (PK_e) to be meaningless? The answer to that question, naturally, depends on the concept of truth involved. Even if we only roughly distinguish between an epistemic and a non-epistemic notion it should be clear that (PK_e) *as such* does not have a determinate meaning; there are at least two different claims that have to be discussed separately ('truth_{ep}' stands for an epistemic concept of truth, 'truth_{¬ep}' for a non-epistemic concept):

(PK_e1) All truths_{ep} are knowable

(PK_e2) All truths_{¬ep} are knowable

As an epistemic concept of truth *is* a notion according to which there is a *conceptual connection* between truth and knowability, (PK_e1) is, obviously, analytically true, and the fact that (PO₃) as well as (PK) are trivial according to Heyting semantics just reflects that intuitionism is a logic that is based on notions like 'proof', 'construction' or 'solution of a problem', i.e. on an epistemic concept of truth. (PK_e2), on the other hand, *presupposes* that a non-epistemic notion of truth is at least coherent, and as intuitionists will deny just that, it is no wonder that it is actually impossible to formalize (PK_e2) by means of a logic that A. Heyting (1956: 228) in his later writings called a "logique du savoir", a logic of knowledge. In intuitionism there is, in fact, no place for the idea of truths that are in principle beyond our cognitive reach.

If what I have argued is sound, two important consequences can be drawn. Firstly: because Fitch's original argument – as well as similar arguments devised more recently (see. e.g. Brogaard & Salerno 2002) – is supposed to be an *immanent* critique

of verificationism it must employ only those logical rules that are acceptable from an anti-realist point of view. If that in turn means that only intuitionistic logic is available as a common formal ground and if intuitionistic operators do have a distinctive meaning, the premises and conclusions of any argument purporting to derive an implausible result from (PK) have to be read according to the BHK interpretation. Thus, even if a sound argument were available that managed to show that (PK) actually implies $(PO_3)^{14}$ there would be no reason for the intuitionist to worry about, as he already accepts the conclusion (PO_3) as a conceptual truth. So the search for alternative versions of the paradox of knowability is quite futile.

The second consequence is similar, but much broader in scope: recently, there have been proposals by J. Salerno (2000) and N. Tennant (2000) to reconstruct certain arguments in the debate on anti-realism by establishing aporias (i.e. inconsistent triads of claims) that all involve, firstly, a thesis that articulates a kind of cognitive limitation on us as knowers and that serves as a common ground; secondly a specifically anti-realist thesis asserting a conceptual connection between truth and knowledge; and thirdly a specifically realist thesis expressing a determinacy of truth-value. According to Salerno's original proposal for the construction of such an aporia (Salerno 2000: 219; Tennant 2000: 846) we may, for instance, derive a contradiction from an epistemically strengthened version of the law of excluded middle (as a realist thesis)

$$(K\text{-LEM}) \quad K(\forall p) (p \vee \neg p),$$

a likewise strengthened version of (PK) (as an anti-realist thesis)

$$(K\text{-PK}) \quad K(\forall p) (p \supset \Diamond Kp),$$

and a claim Salerno calls 'Epistemic Modesty' (as common ground)

$$(EM) \quad \neg K(\forall p) (\Diamond Kp \vee \Diamond K\neg p).$$

¹⁴ See e.g. Rosenkranz (2004: 69) for an interesting suggestion

Assuming – controversially – that the operator 'K' is closed under deducibility, it is easy to show that (K-LEM) and (K-PK) jointly entail the claim

$$(\neg\text{EM}) \quad K(\forall p) (\Diamond Kp \vee \Diamond K\neg p).$$

($\neg\text{EM}$), of course, directly contradicts (EM), and thus an aporia seems to have been successfully established:

$$(K\text{-LEM}), (K\text{-PK}), (EM) \vdash \perp$$

Now Tennant and Salerno explicitly demand that, in order to avoid 'begging any questions in favour of the realist' (Tennant 2000: 826), only intuitionistically valid rules of inference should be employed in the derivation of the contradiction, and they are certainly right about that: as their proposals aim at clarifying arguments for (or against) logical revisionism, one cannot employ a logic in which one of the controversial principles, namely the law of excluded middle, admits of being proved straightforwardly and is true in virtue of the underlying semantics.

But intuitionistic systems cannot serve as a common formal ground either. Although the three theses are in fact constructively (as well as classically) inconsistent:

$$(K\text{-LEM}), (K\text{-PK}), (EM) \vdash_{\text{IC}} \perp,$$

one should, yet again, remember that a switch from a classical to an intuitionistic language substantially affects the meaning of any claim containing logical vocabulary. If the disputants agree to use an intuitionistic language, (K-PK) is not a controversial anti-realist thesis, but becomes part of the common ground.¹⁵ So it seems as though realists and anti-realists trying to solve their dispute by means of a formal language are using the same *symbols*, but not speaking the same *language*. This *problem of shared content* is explicitly addressed by Salerno (2000: 221):

¹⁵ If (PO₃) can be regarded as an uncontroversial conceptual truth (PK) should be such a truth as well, and it is reasonable to assume that conceptual truths are known (by somebody and at a certain point of time).

To the extent that both parties debate only using patterns of inference that both parties accept, their preferred semantics will not mark a difference [...] that makes a relevant difference. It will not make a relevant semantic difference, since in this context their inferential behavior will not reflect one semantics at the cost of another.

Salerno is certainly right in claiming that, as long as the rule of double negation is not used, the 'inferential behaviour' of classicists and intuitionists will not *reflect* any specific semantics. But absent any additional argument, this observation just amounts to requesting that the meaning of the theses be *ignored*. Focussing on the rules of deduction and disregarding the meaning of connectives may enable the disputants to *pretend* that they attach the same meaning to the symbols, but ignoring ambiguities is scarcely ever a good idea. If theses like (K-PK) are not be regarded as mere chains of symbols devoid of any determinate meaning, they have to be interpreted according to either classical or intuitionistic (or maybe some other kind of) semantics; a classical *proof* not using the rule of double negation is not identical to an equiform intuitionistic proof.

This, of course, does not yet imply that realists and anti-realists are inevitably and necessarily talking past one another, for realists and anti-realists *do* agree on a certain set of rules of inference and there *might* be a way to construct a semantics – neither classical nor intuitionistic – on which both parties could agree. It should be clear, however, that in the realism/anti-realism debate – a debate that essentially deals with the justification of deduction and the question “which logic is the right logic?” – neither classical nor intuitionistic languages can serve as an uncontested common ground, and this is also the underlying reason why the realist cannot prove by means of Fitch's argument that anti-realism is incoherent.¹⁶

Thorsten Sander

Universität Duisburg-Essen

thorsten.sander@uni-due.de

¹⁶ I would like to thank an anonymous referee of this journal for several helpful comments on an earlier draft of this paper.

References

- Artemov, S. N. (2001) 'Explicit Provability and Constructive Semantics', *Bulletin of Symbolic Logic*, 7, 1-36.
- Brogaard & Salerno (2002) 'Clues to the paradoxes of knowability: Reply to Dummett and Tennant', *Analysis*, 62, 143-150.
- Cozzo, C. (1994) 'What Can We Learn from the Paradox of Knowability?', *Topoi*, 13, 71-78.
- van Dalen, D. (1986) 'Intuitionistic Logic', in Gabbay & Guenther (eds.) *Handbook of Philosophical Logic*, Vol. III, Dordrecht: Reidel.
- Dummett, M. (1973) 'The Philosophical Basis of Intuitionistic Logic', in *Truth and Other Enigmas*, Cambridge (Mass.): Harvard UP 1978.
- Dummett, M. (1977) *Elements of Intuitionism*, Oxford: Clarendon Press.
- Dummett, M. (2001) 'Victor's Error', *Analysis*, 61, 1-2.
- Fitch, F. B. (1963) 'A Logical Analysis of Some Value Concepts', *Journal of Symbolic Logic*, 28, 135-142.
- Hart, W. D. (1979) 'The Epistemology of Abstract Objects: Access and Inference', *Proceedings of the Aristotelian Society*, Suppl. Vol., 53, 153-165.
- Heyting, A. (1956) 'La conception intuitionniste de la logique', *Les études philosophiques*, 11, 226-233.
- Heyting, A. (1971) *Intuitionism: An Introduction*, Amsterdam: North-Holland Publishing Company.
- Martino & Usberti (1994), 'Temporal and Atemporal Truth in Intuitionistic Mathematics', *Topoi*, 13, 83-92.
- Melia, J. (1991) 'Anti-Realism Untouched', *Mind*, 100, 341-342.
- Raatikainen, P. (2004) 'Conceptions of Truth in Intuitionism', *History and Philosophy of Logic*, 25, 131-145.
- Rasmussen & Ravnkilde (1982) 'Realism and Logic', *Synthese*, 52, 379-437.
- Rosenkranz, S. (2004) 'Fitch Back in Action Again?', *Analysis*, 64, 67-71.
- Salerno, J. (2000) 'Revising the Logic of Logical Revisionism', *Philosophical Studies*, 99, 211-227.

- Shapiro, S. (1993), 'Antirealism and Modality', in Czermak (ed.), *The Philosophy of Mathematics: Proceedings of the 15th International Wittgenstein-Symposium*. Vienna: Hölder-Pichler-Tempsky.
- Tennant, N. (1997) *The Taming of the True*, Oxford: Clarendon Press.
- Tennant, N. (2000) 'Anti-realist Aporias', *Mind*, 109, 825-854.
- Williamson, T. (1982) 'Intuitionism Disproved?', *Analysis*, 42, 203-207.
- Williamson, T. (1988) 'Knowability and Constructivism', *The Philosophical Quarterly*, 38, 422-432.
- Williamson, T. (1992) 'On Intuitionistic Modal Epistemic Logic', *Journal of Philosophical Logic*, 21, 63-89.
- Williamson, T. (2000) *Knowledge and Its Limits*, Oxford: Oxford UP.
- Wright, C. (1987) *Realism, Meaning and Truth*, Oxford: Blackwell.