F. EUGENI, R. MASCELLA, D. PELUSI
Teramo, ITALY

Uncertainty from philosophical and mathematical point of view

1. Introduction

All logic instruments and tools in possession of, and used by researchers are generally considered as the results of bivalent logic. A common error to people interested in science is that, usually, they don't known with certainty which things are true and which are false. But they are sure that things are true or false. No ways in the middle.

The fuzzy principle asserts that this is completely a question of measure. Fuzziness is the opposite concept to bivalency, while fuzzy quality means polyvalency. Fuzzy logic is able to manage uncertain knowledge and, more precisely, to manage those rules difficult to formalize. In many cases, such rules haven't got, or cannot have, the necessary precision, required for a whole formalization.

2. Reasoning and fallacies

The reasoning is a process by means of which human beings put in action their thought, and particularly their abstract thought. One of the more important tasks in logic is, therefore, to explain correct methods of the thought.

The reasoning concept can be regarded like a set of proposal that stand independently one from the others, and takes into account the temporal order of involved events.

In general, the validity (or non-validity) of a reasoning is a different matter with respect to truth or falseness of the proposals it involves. Sometimes, these proposals can be set up in an effective psychological manner: in fact, as is well known, feelings and affections play an important role in the personality of each individual. Thus, a person who converse with others using a sentimental component may have a greater convincing capability than other persons who employ, exclusively, an intellectual component. In this context, the meaning of "fallacy" grows up: it can be defined as an argumentation whose premises don't supply the necessary support in order to get its pretended conclusion.

A deductive fallacy refers to a not valid deductive argumentation. In other words, a fallacious reasoning may have an entire set of true premises and, therefore, conclude the deductive process with a false conclusion. For example, if we consider the premises "If Milan is the capital of Italy, then it is in Italy" and "Milan is in Italy", and the conclusion "Milan is the capital of Italy", we are in presence of a deductive error. In fact, Milan is in Italy, but Italian capital is Rome. Milan is a principal city in Italy, but doesn't entail the conclusion.

Instead, an inductive fallacy is less formal than the deductive one. It is simply a reasoning that seems an inductive reasoning, but the premises don't give enough support to the conclusions. In these cases, even if the premises were true, the conclusions would not acquire greater possibility in being true. For example, "Just arrived in Italy, I have seen a black swan". Conclusion "All italian swans are white.". In this case, we have an inductive error because "Also if there are many swans in Italy, the ones who are black are very few".

It also exists some fallacies called "informal fallacies of ambiguity". In general, the ambiguity concept, intended as a plurality of senses, is an intrinsic phenomenon in linguistic description, in textual argumentation and in communication between individuals. The linguistic patrimony of any speaking person is distinct from individual to individual, so the meaning given by the sender could be different from what interpreted by the receiver. Then, the informal fallacies of ambiguity are connected with a large variety of senses or with the vagueness of the terms used in presenting the argument. The mechanism is very simple to apply: we just need to search a term having more than one meaning and use it as it had only one meaning. For example: "Exceptional performance of the athlete, who had launch the javelin reaching 80 meters. He won a medal. Exceptional strike". "Inflation arrested. In search of its accessories."

3. Fuzzy logic, machines and QI

In strict sense, fuzzy logic is an extension of the multiple values logic, which would be necessary as the logic of approximated reasoning. In a more wide sense, fuzzy logic is a synonym of fuzzy sets theory, that is, the theory of classes with indistinct contours.

The theory of fuzzy sets constitutes an extension of classical sets theory, as in fuzzy theory we cannot apply the aristotelian principles of noncontradiction and of the third excluded.

As a matter of fact, the bivalence sacrifices the accuracy in order to get more simplicity. The mathematical and computer science procedures are simplified by means of binary results of yes and no, of white and black, of true and false. Fuzzy theory extends the traditional sets theory because it doesn’t consider aristotelian ties, it gives liberty where usual logic uses such principles. Thus, fuzzy logic carries us out of the rigid world of zeros and ones: the bits are replaced by fits. A fit value is the fuzzy unit of measure and it can assume all the values between 0 and 1. Respect to bits, the fits represent a soft language, a more adapted way to describe the “gray” world.

The fuzzy approach increases the QI of machines rendering the automation systems closer to human mode of reasoning. In fact, fuzzy systems store tens, hundreds or thousands of fuzzy rules characterized by common sense. Every new data actives all fuzzy rules previously defined in a certain measure. The fuzzy system mix the results and produces at the end an answer.

In conclusion, fuzzy rules correlate fuzzy concepts creating several combinations. This allows the machines to increase their potential QI: we won’t be anymore in a situation in which the air conditioner blow too much cold or too much hot air, or that corrections in the locking system of trains were executed through clicks.

4. Uncertainness and Uncertainty

The term “uncertainness” is generally opposite to almost any meaning connected to certain matters.

An uncertain event is a particolar situation which is not identifyable under sufficient security margins. A classic example rely in modern economic markets, where economic and strategic decisions on investments have to be taken under uncertain competitive contexts. Nowadays, financial theories supply many instruments to evaluate the quality of investments. Nevertheless, such models bear with certain values of cash flows.

In the most part of real situations, the values of financial flows coming up from investments cannot be determined with certainty. In fact, during their life period, social and economical conditions may change, affecting the aforementioned quality. In financial exchanges, decisions are usually taken trusting on a particular development of the market. But anyone is able to indicate with appropriate precision whether future development can be considered trustable. In conclusion, we have to face matters affected by uncertainty.

Fuzzy logic, from its logic point of view, is able to emulate human reasoning. In fact, it is based on the natural language and, so, it is able to represent a market agent behaviour. Obviously, fuzzy logic cannot pursue the irrealistic objective of eliminating the uncertainty whithin the entire decisional process. However, it supplies a better way to take into account the presence of uncertainty in economical-financial context.

In sciences, the term "uncertainty" is joint to the error made in any process of measurement. For example, no physical quantity can be measured with complete accuracy and certainty. Working carefully, it is possible to reduce uncertainties until they become extremely small. But it is impossibile to eliminate them at all.

All measurements, also if precise and scientific, cannot avoid uncertainties. The analysis of errors is the study, and the evaluation, of such uncertainties. This brings two main aims to the scientists: estimating how much the uncertainties are wide, and helping in reducing them when needed. Moreover, the possibility to estimating and reducing uncertainties to a more appropriate level, in order to make affordable conclusions, may transform a series of boring and ripetitive measurements to an interesting experiment.

In fuzzy systems an important role is played by knowledge basis. This is an extension of the traditional basis of data, in the sense that it is a knowledge repertoire which is pertinent to the domain of the problem to solve. Generally, knowledges are represented through rules. Every rule is constituted by a premise and a conclusion. The conclusion will be true to the extent that the premise is true. Knowledge, pictured through this kind of rules, is certain, deterministic, nonambiguous. Rules can, in principle, express uncertain knowledge by means of certainty factors in the premise and/or in the conclusion. Such factors, far from being interpreted as probabilistic values, give empiric numeric coefficients that quantify the system user confidence on the assertion of the premise and/or on the rule that states the conclusion. Such coefficients can reveal an absolute uncertainty, in case of a zero-value. In all other cases, uncertainty, and certainty, are weighted.

5. Connections with Mathematics

It is a wide opinion, inside scientific community, that fuzzy logic and structures have simply vague likeness with probability structures. In our current state, the use of fuzzy structures within mathematical sciences are, perhaps, to be searched in thematics regarding hyperstructures. In fact, the state of the art of research on hyperstructures has developed a deep tendency on new environments. In here, the basic sets (on which hyperoperation are defined) are not rigid and rigorous, but soft and indistinct in the fuzzy theory conceptualization.

For this aim, it is possible to make reference to P. Corsini and to the journal "Italian Journal of Applied Mathematics". Such review includes many papers in this direction and, from these, is a simple job to find an international bibliography on the subject.

In our opinion, however, and in relation to an epistemological idea of a general theory of the uncertainty, it should be investigated the possibility to build connections between the philosophies of fuzzy theory and subjective probability, as it was thought, in the beginning, by B. De Finetti.

In conclusion, it is worth to notice that all the investigations in social studies are characterized by uncertainty attitudes which are very hard to rule from the quantitative point of view. This is just the problem on which we would like to open a debate.

6. Conclusion

Logic is the science who teaches people to use reasoning, or to deduct from known and determined propositions, some other propositions. From a conceptual point of view, as De Finetti stated: "there is no difference between logics and mathematics". When we need to make complex reasonings, mathematics becomes a sharp logic tool. Mathematics assume a greater importance if we need to obtain precise results from precise data. On the other hand, physics teaches us that neat numerical values are inexistent: in any measurement, it always exists an uncertainty associated to it. In this sense, we cannot consider them as true values, but simply as the best values associated to a measurement.

Thus, our world exibits real situations whose formalization cannot be complete, and whose rigorousness degree is hardly attenuated. Fuzzy logic finds his proper location in areas where we need to create very simple models which are close to human reasoning. And, thus, without complex mathematical equations.

In order to analyze the real world, where uncertainty is widespread, fuzzy logic is particularly suitable. In fact, simulation by means of fuzzy models is widespread in systems which give support to decisions, for generating and analyzing possible sceneries. Fuzzy inference engine is built and tared for emulating the behaviour of a complex system, as the socio-economic system of a community. Sometimes, fuzzy algorithms are used with diagnostic and future prediction intents. In such cases, they are able to reconstruct informations starting from uncertain data.

It is important to remark that the degree of belonging of a fixed element to a fuzzy set is not a probability. This is well known. When we say that a glass has 50% of probability to be full, we predict to observe 50 events out of 100 in which such glass is full. Instead, when we state that the degree of belonging of a determined glass to a fixed fuzzy set is equal to 0.5, we state that its fullness has got an ambiguity degree of 50%. In the first case, we have the measure of the statistic frequency of a certain event; in the second, we have the measure of a deterministic, nonambiguous fact.

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